# Analysis of a Radioisotope Thermal Rocket Engine

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## Abstract

The Triton Hopper is a concept for a global hopper vehicle which uses a radioisotope rocket engine and In-situ propellant acquisition to explore the surface of Neptune's moon, Triton. The current Triton Hopper concept stores heated Nitrogen in a spherical tank to be used as the propellant. The aim of the research was to investigate the benefits of storing propellant at ambient temperature and heating it through the use of a thermal block during engine operation, as opposed to storing gas at a high temperature. Lithium, Lithium Fluoride and Beryllium were considered as possible materials for the thermal block. A heat energy analysis indicated that a lithium thermal mass would provide the highest heat energy for a temperature change from $900^0$C to $-100^0$C. A heat transfer analysis was performed for Nitrogen at $-100^0$C flowing through 1000 passages inside a 1kg lithium thermal block at a temperature of $900^0$C. The system was analyzed as turbulent flow through a tube with constant surface temperature. The analysis indicated that the propellant reached a maximum temperature of $877^0$C before entering the nozzle. At this exit temperature, the average specific impulse ($I_{sp}$) of the engine was determined to be 157s. Previous studies for the stored heated gas concept suggest that the engine would have an average $I_{sp}$ of approximately 52s. Thus, the use of a thermal block concept results in a 200% engine performance increase.

In addition, a tank sizing study was performed to determine if the concept is feasible in terms of mass requirements. The mass for a spherical carbon fiber COPV storing 35kg of nitrogen at an initial temperature of $-100^0$C and a pressure of 1000psia, was determined to be 7.2kg. The specific impulse analysis indicated that the maximum engine performance is obtained for a mass ratio of 5kg of Nitrogen per every 1kg of lithium thermal mass. Thus for 35kg of Nitrogen the total thermal mass would be 7kg. This brings the total mass of the system to 49.2.kg which is less than the 56kg landing payload capacity of the Triton Hopper. Finally, an insulation analysis using 10mm of MLI insulation indicated that a total of 22 watts of heat are lost to the environment. With the heat loss known, the power required to heat the thermal mass to $900^0$C in 24 days was determined to be 2.15 watts.

The study's results allowed us to conclude that the thermal mass concept is the better option due to the performance increase provided, the low power requirement and its compliance with the landing mass requirement of the Triton Hopper.

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# I. Introduction

Triton Hopper is a conceptual design for a Global Hopper that will explore Neptune's largest moon Triton. This celestial body was visited once by Voyager 2 in 1989, but it was only a fly-by mission. Triton is the only large moon in the solar system with a retrograde orbit and because of this and its surface composition, it is believed to be a captured object from the Kuiper belt. The Triton Hopper will use In-situ propellant acquisition to allow for the exploration of a large area of Triton. It will gather frozen nitrogen from Triton's surface and store it in a tank for use as propellant. Triton Hopper will be powered by a radioisotope power system which will heat the propellant, and provide thermal support to other components. The Triton Hopper would be the first ever vehicle to explore a Kuiper belt object. In addition, a vehicle that utilizes frozen gases as propellant would be revolutionary and would be a pathfinder for later designs to explore other celestial bodies.

The current propulsion system proposed consists in storing heated nitrogen in a tank. The gas would then be expelled through the nozzle. The biggest disadvantage of the system is that it requires heavy pressurized gas tanks to store the heated nitrogen. In addition, the nitrogen cools as the tank blows down which results in a decrease in performance. The new system proposed consists of storing cold nitrogen in a spherical tank and passing it through a thermal block during engine operation, to heat the propellant to a desired temperature before it exits through the nozzle. Heating the gas as it flows through the system, allows for higher gas temperatures at the exit which provides a higher specific impulse.

The biggest challenge of the new concept is designing a system that provides enough heat to raise the nitrogen's temperature from -100°C to 900°C, while also being light enough to meet the mass requirements of the mission. A system like the one proposed requires a high heat storage and high heat transfer rate. Lithium, Lithium Fluoride and Beryllium were the materials considered for use as the thermal mass. The following report presents the results for a heat storage capacity analysis and specific impulse analysis for the different thermal mass materials. The material that provides the best performance will be used to run a heat transfer analysis for the thermal block concept proposed. The purpose of the analysis is to determine if the material provides the high heat transfer rate needed to allow the propellant to reach the 900°C exit temperature. An insulation analysis will also be performed in order to determine the amount of heat lost by the system. This will allow us to determine the amount of Radioisotope Heater Units (RHUs) or General Purpose Heat Source modules (GPHS) needed to heat the thermal mass.

# II. Thermal Block Material Analysis

The radioisotope engine concept proposed requires a high heat storage capacity, and high heat transfer rate to achieve the desired exit temperature of the propellant. Lithium, Lithium Fluoride, and Beryllium were considered as possible thermal block materials. Lithium provides the advantage of a high heat capacity in both solid and liquid state, while also being the lightest of the materials considered. Lithium Fluoride is a phase change salt which provides a high heat capacity while also providing a high heat of fusion at its melting point of 848°C. Beryllium was considered because of its high heat capacity and the fact that it stays solid for the entire operation temperature range of the thermal block. A material such as Beryllium would eliminate the heat transfer reduction problems caused by the solidification of phase change materials used for the thermal block.

The following sections present the calculations for heat, and specific impulse ($I_{sp}$), that ultimately led to the selection of Lithium as the best material for the thermal block of the radioisotope engine.

## A. Heat Energy
The thermal energy released or absorbed per mass of substance is given by equation 1.

$$\dot{Q} = C_p \Delta T \tag{1}$$

Where $\dot{Q}$ is the amount of heat generated per mass of propellant (kJ/kg), $C_p$ is the heat capacity of the material (kJ/kg-°C) and $\Delta T$ is the change in temperature (K). The heat energy was calculated for a temperature change from -100°C to 900°C. The steps in Fig.1 where the heat increases abruptly are due to the release of the heat of fusion.
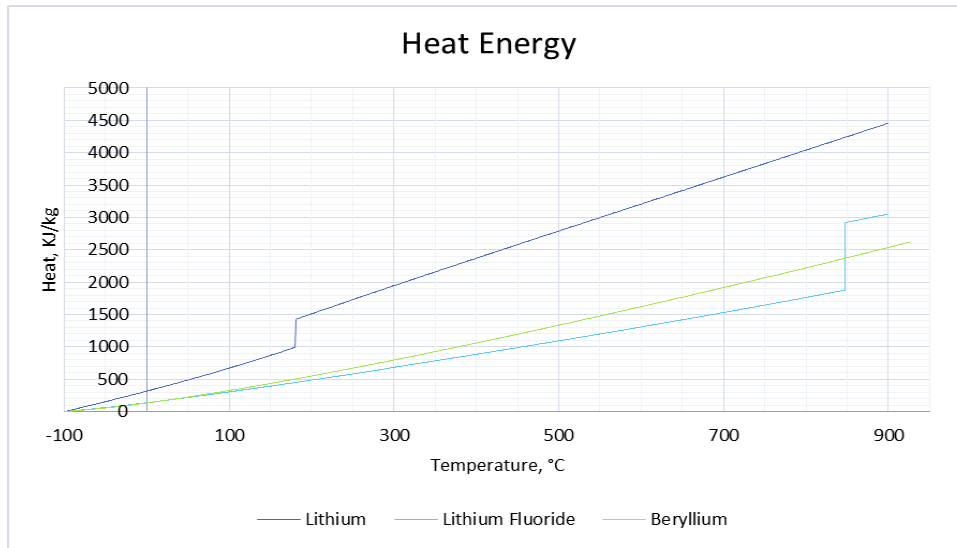
**Figure 1. Heat energy of thermal block materials.** *The plot shows the amount of heat released or absorbed by the materials for a temperature range from -100°C to 900°C.*

## B. Specific Impulse

Specific impulse ($I_{sp}$) describes how much thrust is delivered by an engine per propellant mass flow rate (Brown, 2002). It is a measure of the efficiency of the engine and is given by equation 2.

$$I_{sp} = \frac{V_e}{g} \tag{2}$$

Where $I_{sp}$ is the specific impulse (s), $V_e$ is the exhaust velocity (m/s), and g is the gravitational constant (m/s$^2$).

The exhaust velocity of the engine is determined from equation 3.

$$V_e = \sqrt{\frac{2\,k\,R\,T_c}{k-1} * \left[1 - \left(\frac{P_e}{P_c}\right)^{\frac{k-1}{k}}\right]} + \frac{P_e A_e}{P_c A_t} * \sqrt{\frac{R\,T_c}{(k g_c \left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}} \tag{3}$$

Where $V_e$ is the exhaust velocity in m/s, k is the ratio of specific heats of the gas, R is the ideal gas constant (J/K-mol), $T_c$ is the chamber temperature (K), $P_e$ is the absolute pressure at the exit (Pa), $P_c$ is the absolute chamber pressure (Pa), $A_e$ is the cross sectional area of the nozzle exit in m$^2$, and $A_c$ is the cross sectional area of the throat (m$^2$). Assuming that the pressure at the exit of the nozzle is the same as the pressure of the chamber, the exhaust velocity can be expressed as:

$$V_e = \sqrt{\frac{2\,k\,R\,T_c}{M(k-1)}} \tag{4}$$

Where M is the molecular weight of the gas, which is 26.98 g/mol for Nitrogen.

The first step in determining the impulse generated by the engine is to determine the amount of mass that needs to flow through 1kg of thermal mass in order to reduce its temperature by 1°C. This is given by equation 5.

$$\Delta m_{N2} = C_p m_{tm}/\dot{Q}_{N2} \tag{5}$$

Where Δm is the mass of Nitrogen in kg, $C_p$ is the heat capacity of the heat reservoir material (kJ/kg-°C), $m_{tm}$ is the mass of the thermal mass (kg), and $\dot{Q}_{N2}$ is the amount of heat per kilogram released or absorbed by Nitrogen for a change in temperature. The impulse generated by the engine is determined by multiplying the exhaust velocity of the propellant by the mass of nitrogen required to reduce the temperature of 1 kilogram of material by 1 degree Celsius.

$$I = w_p g I_{sp} = w_p V_e \tag{6}$$

The plot in Fig.2 show the impulse generated by the engine for the different thermal block materials proposed.
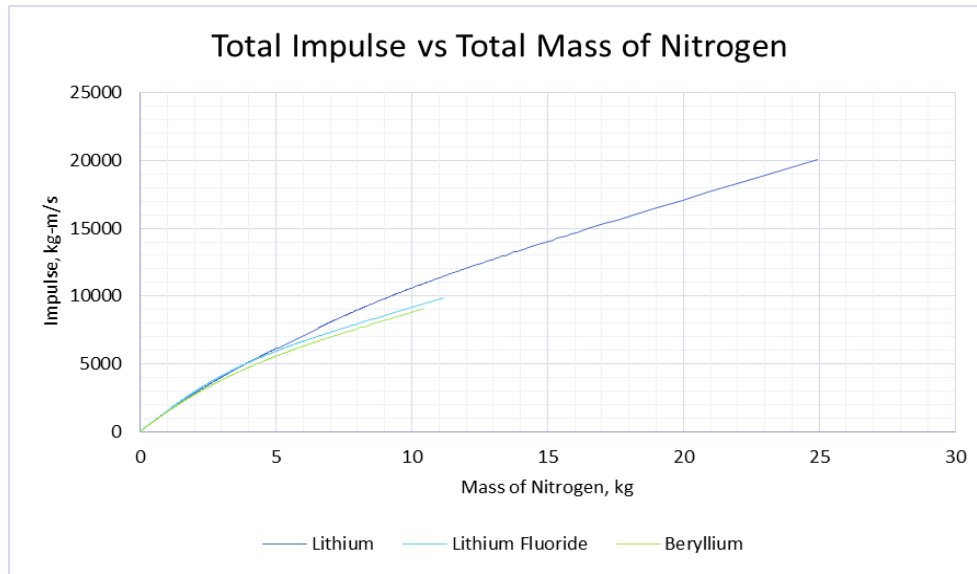


**Figure 2. Total impulse vs. mass of nitrogen.** *The plot show the total impulse generated by the engine as a function of the amount of propellant mass that needs to flow through the 1kg of thermal mass.*

The average specific impulse is defined as;

$$I_{sp_{avg}} = (I/m_{N2})/g \tag{7}$$

Where I is the total impulse generated by the engine in kg-m/s, $m_{N2}$ is the mass of Nitrogen (kg), and g is the gravitational constant (m/s²). The plot shown in Fig.3 presents the average $I_{sp}$ generated by the engine for a change in temperature from -100°C to 900°C. The results indicate that the engine which uses a thermal block made of lithium would provide the best performance.
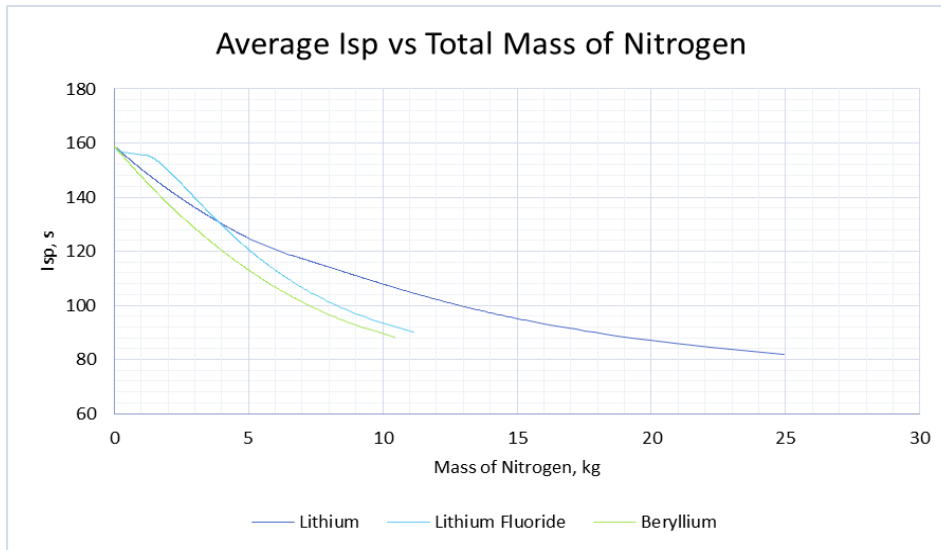
**Figure 3. Average specific impulse.** *The plot shows the average $I_{sp}$ generated by the engine for the different thermal block materials proposed.*

The effective specific impulse, takes into account the weight of the thermal mass and is a more accurate representation of the engine's performance. The plot shown in Fig.4 presents the specific $I_{sp}$ generated by the engine for a change in temperature from $-100^0C$ to $900^0C$.

$$I_{sp_{Eff.}} = [I/(m_{N2} + m_{tm})]/g \qquad (8)$$
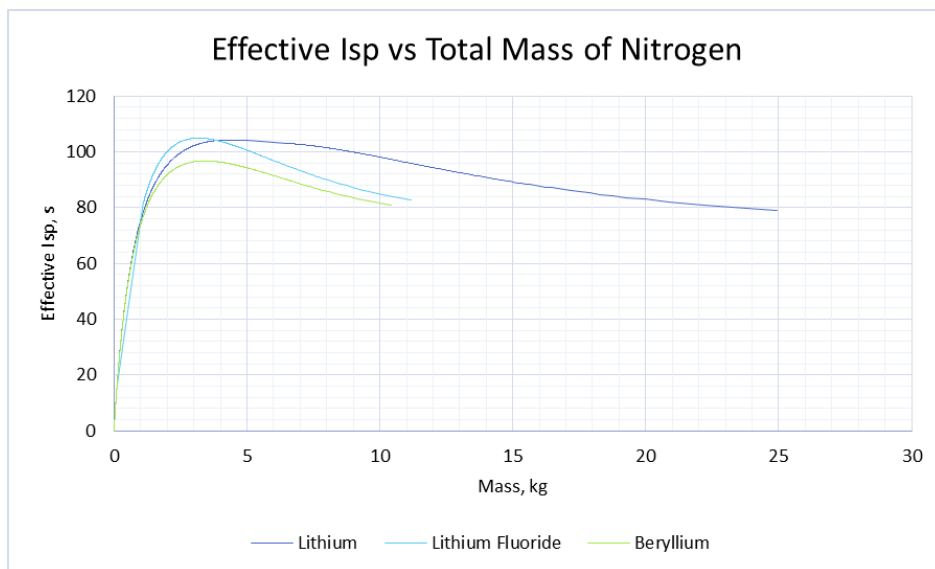


**Figure 4. Effective specific impulse.**

# III.   Heat Transfer Analysis

The system proposed consists of a thermal block which acts as a heat exchanger with passages through which the propellant flows. As the propellant flows through the passages, it heats up to a temperature close to 900°C before exiting through the nozzle. A preliminary heat transfer analysis was performed to determine if the heat reservoir system would produce the heat transfer rate needed to achieve the 900°C exit temperature of the propellant. The system was analyzed as heat transfer through a tube with constant surface temperature. It was assumed that the temperature of the reservoir material remains constant throughout the process, thus providing the constant surface temperature condition. In addition, heat transfer through conduction was ignored. The values of the properties of the propellant are for nitrogen at atmospheric pressure. For preliminary analysis purposes it is assumed that these properties do not change with pressure.

## A.  Heat transfer analysis theory

For a flow inside a tube which has constant surface temperature, the heat transfer to or from the fluid is given by Newton's law of cooling.

$$\dot{Q} = hA_s\Delta T_{avg} = hA_s(T_S - T_m)_{avg} \tag{9}$$

Where h is the convective heat transfer coefficient in W/m²-°C, $A_s$ is the heat transfer surface area of the tube which is equal to $\pi DL$ (m²), and $\Delta T_{avg}$ is the average temperature difference between the fluid and the surface of the tube.

The log mean temperature was used as the average temperature difference. The log mean temperature is an exact representation of the average temperature difference between the fluid and the surface temperature, which is why it is used to determine the properties of the fluid and to calculate the heat transfer rate (Çengel & Ghajar, 2015).

$$\Delta T_{lm} = \frac{T_i - T_e}{\ln[(T_s - T_e)/(T_s - T_i)]} \tag{10}$$

Where $T_i$ is the initial temperature of the fluid at the tube entry, $T_s$ is the surface temperature of the tube, and $T_e$ is the exit temperature of the fluid at the tube exit.

The convective heat transfer coefficient depends on whether the fluid flow is laminar or turbulent. This is determined from the Reynolds number. For flow inside a tube, a Reynolds number smaller than 2,300 means that the fluid is laminar, while a number larger than 4,000 means the flow is fully turbulent. Any number in between 2,300 and 4,000 means the fluid is in its transition state between laminar and turbulent. The Reynolds number is defined by the following function:

$$Re = \frac{\rho V_{avg}D}{\mu} = \frac{4\dot{m}}{\mu \pi D} \tag{11}$$

Where $\mu$ is the viscosity of nitrogen at the mean log temperature, and D is the diameter of the tube. The density of nitrogen ($\rho$) is calculated by dividing the mass of gas being stored by the volume of the tank (kg/m³).

The heat transfer coefficient can be determined from the Nusselt number (Nu) which is defined as:

$$Nu = \frac{hD}{k} \tag{12}$$

Where h is the convective heat transfer coefficient, D is the diameter of the tube (m), and k is the heat conduction coefficient of the fluid (W/m-°C). For laminar fluid flow in a tube of constant surface temperature, the Nusselt number has a value of 3.66. For turbulent flow, the Nusselt number is a function of the Reynolds number (Re) and the Prandtl number (Pr), and is defined as:

$$Nu = 0.023\,Re^{0.8}Pr^n \tag{13}$$

Where n is 0.4 for heating and 0.3 for cooling of the fluid.

For turbulent flow, the Nusselt number is given by the Gnielinski equation.

$$Nu = \frac{(f/8)(Re - 1000)Pr}{1 + 12.7(f/8)^{\frac{1}{2}}\left(Pr^{\frac{2}{3}} - 1\right)} \tag{14}$$

The Prandtl number is defined as:

$$Pr = \frac{\mu C_p}{k}$$ (15)

Where $\mu$ is the viscosity in kg/m-s, $C_p$ is the heat capacity (kJ/kg-$^0$C), and k is the thermal conductivity of nitrogen at the mean log temperature (W/m-$^0$C).

The average velocity of the fluid is determined from the mass flow rate of nitrogen.

$$V_{avg} = \dot{m}/\rho A_c$$ (16)

Where Ac is the cross sectional area of the tube (m$^2$). The case being analyzed is for 1 tube of the thermal block, thus the mass flow rate used in the analysis is the mass flow rate of Nitrogen entering the thermal block, divided by the amount of passages in it.

The temperature of the propellant at the exit of the tube is given by equation 17 for flow through a tube with constant surface temperature.

$$T_e = T_s - (T_s - T_i) \exp\left(-\frac{hA_s}{\dot{m}C_p}\right)$$ (17)

The temperature difference between the surface of the fluid decays exponentially as the magnitude of the exponent increases. The value of the exponent is known as the number of transfer units, denoted by NTU. The unitless number NTU is a measure of the effectiveness of the heat transfer system. An NTU value of 5 or higher means the heat transfer rate has reached a maximum and assures that the exit temperature of the fluid will be almost equal to the surface temperature of the tube. A value of less than 5 means the heat transfer can be improved by extending the length of the passage.

**B. Pressure loss theory**

The pressure loss at the exit of the tube is given by equation 18. Where $\rho V_{avg}^2/2$ is the dynamic pressure and $f$ is the Darcy friction factor. The $\left(\frac{T_i + T_o}{2T_i}\right)$ factor in the equation accounts for the fact that compressible fluids expand due to the pressure drop.

$$\Delta P_L = f\frac{L}{D}\left(\frac{\rho V_{avg}^2}{2}\right)\left(\frac{T_i + T_o}{2T_i}\right)$$ (18)

The value of the Darcy friction factor depends on whether the flow is laminar or turbulent. For laminar flow the Darcy friction is given by $f = 64/Re$, whereas for turbulent flow in a smooth tube, the Darcy friction factor is given by $f = (0.790 * \ln(Re) - 1.64)^2$.

**C. Heat transfer analysis for a lithium thermal block**

A specific case was studied for a heat reservoir composed of 1000 tubes of 1 millimeter diameter and 20 centimeters in length. The heat transfer analysis was performed for 1 of the 1000 tubes. A total mass of 35 kilograms of nitrogen was assumed to be stored in a spherical tank at a pressure of 1000psi (6894.76kPa). The mass flow rate of nitrogen through the thermal block was assumed to be 100 grams per second. The temperature of the stored nitrogen was assumed to be -100$^0$C, and the surface temperature of the tube was assumed to be a constant 900$^0$C. In order to calculate the mean log temperature, the temperature at the exit of the tube was assumed to be 899$^0$C, meaning that the fluid reaches the temperature of the surface of the tube. All the properties of nitrogen were evaluated at the mean log temperature given by equation 10.

$$\Delta T_{lm} = \frac{T_i - T_e}{\ln[(T_s - T_e)/(T_s - T_i)]} = \frac{-100 - 900}{\ln[(900 - 899)/(900 + 100)]} = 145^0C$$

The heat capacity, viscosity, and heat conductivity of nitrogen at 145 $^0$C are:

$$C_p = 1.0458 \frac{kJ}{kg°C}$$

$$\mu = 2.267 \times 10^{-5} \frac{kg}{ms}$$

$$k = 0.0339 \frac{W}{m°C}$$

The density of nitrogen was determined from the assumption that 35kg are being stored in a spherical tank at a pressure of 1000psi (6894.76kPa). The volume of the tank is determined from the ideal gas law.

$$PV = nRT \rightarrow V = \frac{nRT}{P} = \left( \frac{35kg}{0.028 \frac{kg}{mol}} \right) * \left( 8.3145 \frac{J}{Kmol} \right) * \left( \frac{173K}{6.895 * 10^6 Pa} \right) = 0.261 \, m^3$$

$$\rho = \frac{m}{V} = 134.3 \frac{kg}{m^3}$$

The mass flow rate per tube of the heat reservoir is determined by dividing the total flow rate by the amount of passages, and the average fluid velocity is determined from equations 16.

$$\dot{m} = \frac{\left( 0.1 \frac{kg}{s} \right)}{1000} = 0.0001 \frac{kg}{s}$$

$$A_c = \frac{\pi}{4} D^2 = \frac{\pi}{4} * (0.001m)^2 = 7.85 \times 10^{-7} m^2$$

$$V_{avg} = \frac{\dot{m}}{\rho A_c} = \frac{0.0001 \frac{kg}{s}}{134.3 \frac{kg}{m^3} * 7.85 \times 10^{-7} \, m^2} = 0.95 \frac{m}{s}$$

The Reynolds number is calculated from:

$$Re = \frac{\rho V_{avg} D}{\mu} = \frac{\left( 134.3 \frac{kg}{m^3} \right) \left( 0.95 \frac{m}{s} \right) (0.001m)}{\left( 2.267 \times 10^{-5} \frac{kg}{ms} \right)} = 5615.67$$

Re > 4000 thus, the fluid flow is fully turbulent.

Knowing that the flow inside the tube is fully turbulent, the Prandtl, Nusselt number, and convective heat transfer coefficient are calculated from equations 15, 14 and 12, respectively.

$$Pr = \frac{\mu C_p}{k} = \frac{\left( 2.267 \times 10^{-5} \frac{kg}{ms} \right) \left( 1.0458 \frac{KJ}{kg°C} \right)}{0.0339 \frac{W}{m°C}} = 0.70$$

$$Nu = \frac{(f/8)(Re - 1000)Pr}{1 + 12.7(f/8)^{\frac{1}{2}} \left( Pr^{\frac{2}{3}} - 1 \right)} = \frac{(0.037/8)(5615.67 - 1000) * 0.70}{1 + 12.7(0.037/8)^{1/2} \left( 0.70^{\frac{2}{3}} - 1 \right)} = 18.42$$

$$Nu = \frac{hD}{k} \rightarrow h = \frac{Nu\,k}{D} = \frac{\left(18.42 * 0.0339\,\frac{W}{m°C}\right)}{0.001m} = 625\,\frac{W}{m^2°C}$$

The number of transfer units is determined as follows;

$$A_s = \pi D L = \pi(0.001m)(0.2m) = 0.0006283\ m^2$$

$$NTU = \frac{hA_s}{\dot{m}C_p} = \frac{\left(625\,\frac{W}{m^2°C}\right)(0.0006283\ m^2)}{0.0001\,\frac{kg}{s} * 1.0458\,\frac{KJ}{kg°C}} = 3.75$$

The NTU value is close to 5 which indicates that the temperature of the fluid at the exit will be close to the surface temperature of the tube (Çengel & Ghajar, 2015). Using the equation for the exit temperature and substituting the values for the variables yields;

$$T_e = T_s - (T_s - T_i)\exp\left(-\frac{hA_s}{\dot{m}C_p}\right) = 900\ °C - (900\ °C + 100\ °C)\exp(-3.75)$$
$$T_e = 877\ °C$$

As expected, the exit temperature at the exit is close to the surface temperature of the tube (900°C).

The heat transfer rate to the fluid is defined as;

$$\dot{Q} = hA_s\Delta T_{lm} = \left(625\,\frac{W}{m^2°C}\right)(0.0006283\ m^2)(145\ °C) = 57W$$

**D. Pressure drop**
   Finally, the pressure drop throughout the tube is determined from the pressure loss equation using the Darcy friction factor for turbulent flow inside a smooth tube.

$$f = (0.790\ln Re - 1.64)^{-2} = (0.790\ln(5615.67) - 1.64)^{-2} = 0.037$$

$$\Delta P_L = f\frac{L}{D}\left(\frac{\rho V_{avg}^2}{2}\right)\left(\frac{T_i + T_o}{2T_i}\right) = (0.037)\left(\frac{0.2m}{0.001m}\right)\left(\frac{134.3\,\frac{kg}{m^3} * \left(0.95\,\frac{m}{s}\right)^2}{2}\right)\left(\frac{-100 + 899}{2 * -100}\right) = 1747Pa$$

## IV.  Heat Transfer Analysis

**A. Fluid temperature gradient**
Numerical integration was used to determine the temperature gradient of the fluid along the tube. The fluid exit temperature was analyzed at 1mm length increments of the tube. The surface area and NTU value of each section mas calculated, and the surface temperature of the fluid was determined from equation 17.

$$T_e = T_s - (T_s - T_i)\exp\left(-\frac{hA_s}{\dot{m}C_p}\right)$$

Where the NTU is defined as $-\frac{hA_s}{\dot{m}C_p}$.

   The results obtained are consistent with the ones obtained from the previous analysis. The exit temperature of the fluid at the end of the passage is 877°C. To obtain an exit temperature higher than this would require longer passages. An increase in length of the tubes results in an increase in the NTU number. Once the NTU reaches a

value of 5, the heat transfer has reached its limit and the exit temperature will not increase with any further increase in tube length.
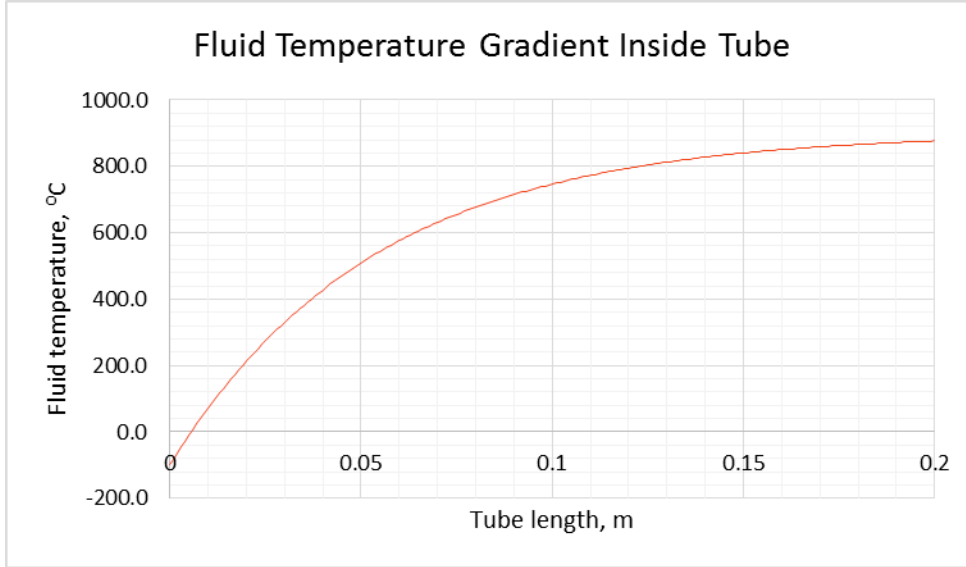
## Fluid Temperature Gradient Inside Tube



**Figure 5. Fluid temperature gradient.** *Plot shows the temperature gradient of the fluid along the tube which has a constant surface temperature of 900⁰C. The fluid enters the thermal block at -100⁰C and exits at 877⁰C.*

The amount of heat added to the fluid with each increment of temperature is determined from equation 19.

$$\Delta \dot{Q} = \dot{m} C_p \Delta T = \dot{m} C_p (T_e - T_i) \tag{19}$$

Where $\dot{m}$ the mass flow rate per tube, $C_p$ is the heat capacity of nitrogen at the mean logarithmic temperature and $\Delta T$ is the temperature change of the fluid between the entry and the exit of the 1mm section being analyzed. The analysis indicates that a total of 102W need to be added to the fluid for it to reach an exit temperature of $877^0$C.

## B. Thermal mass tube surface temperature gradient.

The surface temperature gradient along the tube was analyzed for a thermal mass made out of lithium. Numerical integration with 1mm tube length increments was used to determine the surface temperature gradient along the tube. The amount of thermal mass per section of tube length was determined from equation 20.

$$\Delta m = \frac{m_{th} \Delta x}{L * 1000} = \frac{1kg * 0.001m}{0.2m * 1000} = 0.000005kg = 5\mu g \tag{20}$$

Thus, each 1mm section of tube is surrounded by 5μg of Lithium.

The rate of temperature change of the surface of the tube is determined from equation 21.

$$\frac{dT}{dt} = \frac{\Delta \dot{Q}}{\Delta m C_p} \tag{21}$$

Where $\Delta \dot{Q}$ is the heat added to the 1mm section of tube, $\Delta$m is the amount of Lithium surrounding the section of tube, and $C_p$ is the heat capacity of lithium, evaluated at the temperature of the fluid.
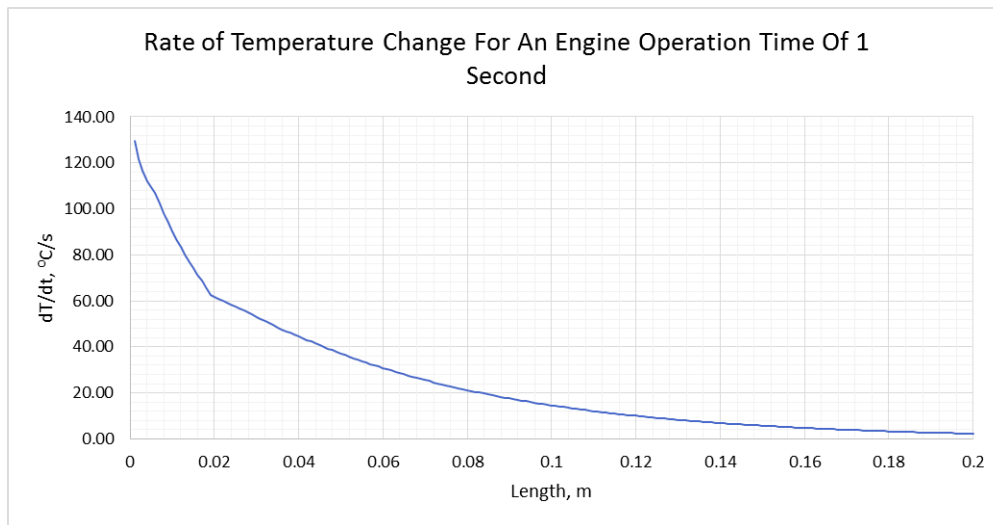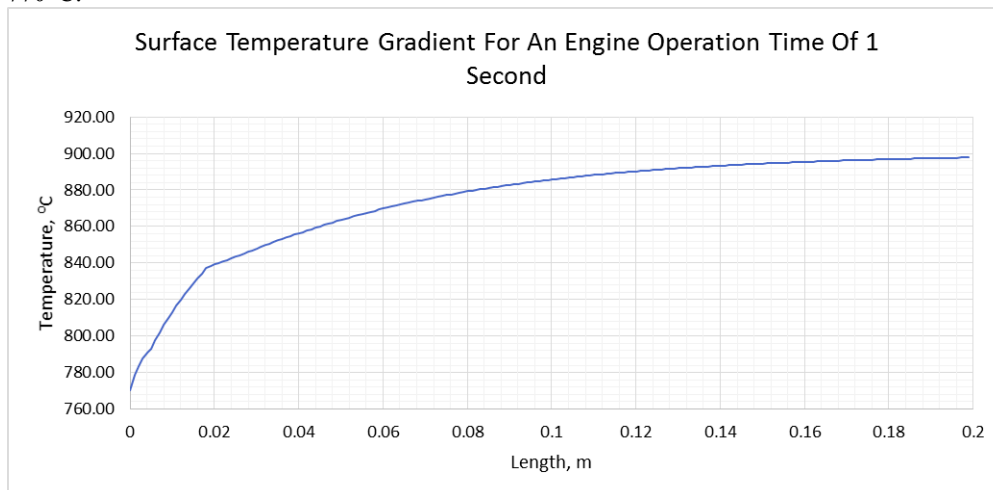
**Figure 6. Rate of temperature for an engine operation time of 1 second.** *The plot shows the rate at which the temperature of the surface of the tube decreases for an engine operation time of 1 second.*

The surface temperature of the tube is determined by subtracting the temperature change resulting from 1 second of engine operation, from the initial surface temperature of the tube.

$$T_{sf} = T_{s0} - t * \frac{dT}{dt}$$

During the first second of engine operation, the surface temperature of the entry region of the passage drops to 770°C.



**Figure 7. Surface temperature gradient for an engine operation time of 1 second.** *The plot shows the change in temperature of the surface of the tube throughout its length for 1 second of engine operation time.*

The temperature of the surface of the tube is changing with time, which means the amount of heat entering the fluid is also changing with time because the thermal mass is cooling down. This means that the rate of temperature change of the surface, the fluid temperature, and the heat transfer, all need to be recalculated for each second of operation. The results for an engine operation time of 50 seconds are presented in figure 8.
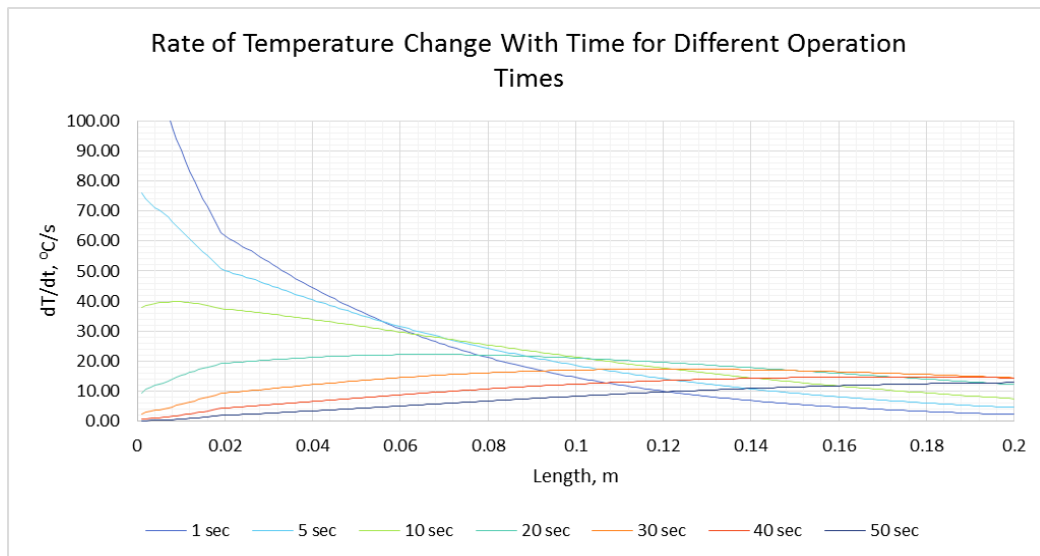
**Figure 8. Rate of surface temperature change for various engine operation times.**
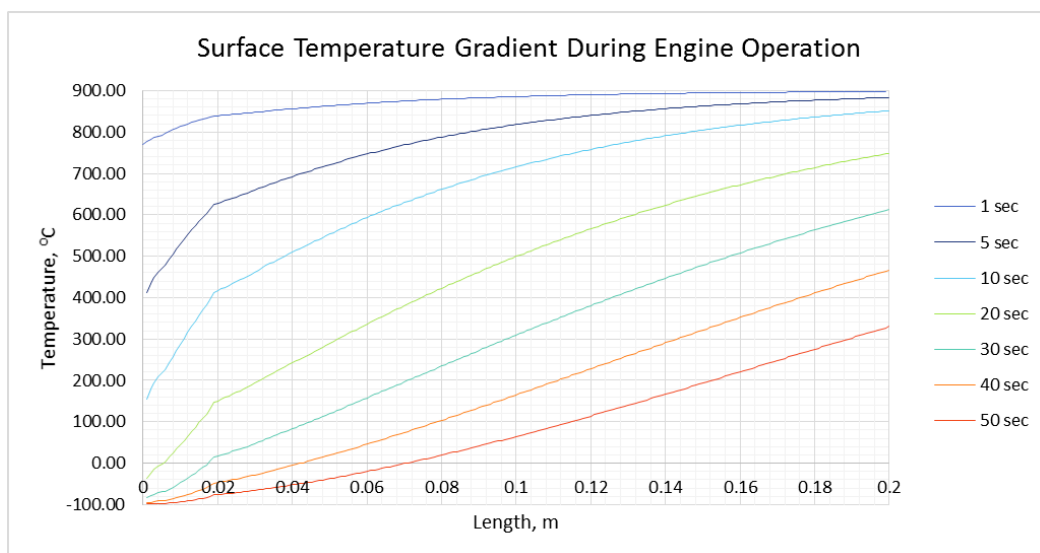


**Figure 9. Surface temperature gradient during various engine operation times.**

## V.   Corrected Specific Impulse

Once the temperature gradient across the tube length is known, the average and effective specific impulse are recalculated to determine how the change in temperature affects the performance of the engine. Since the thermal mass is cooling down as nitrogen passes through it, the performance of the engine will decrease with time. To determine the performance of the engine as a function of engine operation time, the fluid temperature at the exit is calculated for 50 seconds of engine operation. This is done by calculating the fluid temperature at the exit of the passages for 1 second time increments. The change in fluid exit temperature as a function of time is shown in figure 10.
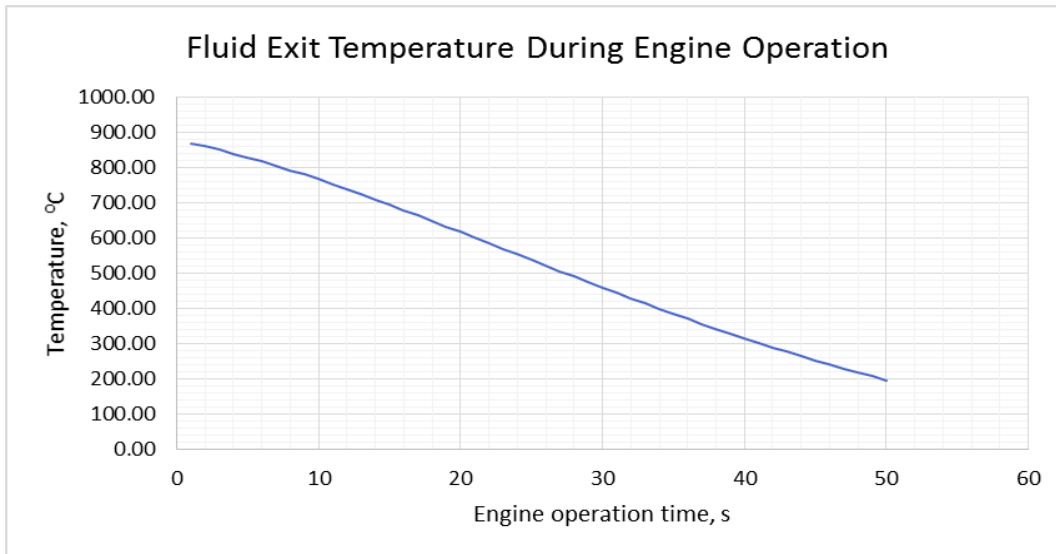
**Figure 10. Propellant exit temperature as a function of engine operation time.**

Next, the exhaust velocity is recalculated with the new values for fluid exit temperature. In the initial calculations for specific impulse, the Δm of nitrogen determined represented the amount of nitrogen needed to reduce the temperature of 1kg thermal mass by $1^0C$. Assuming that the temperature gradient of the thermal mass is equal to the temperature gradient of the surface of the tube, and using a Δm of 0.1kg, which corresponds to the mass flow rate of 0.1kg/s used in the heat transfer analysis, the performance of the engine can be determined for any desired engine operation time. With the exhaust velocity and the Δm determined, the impulse can be calculated from $I = V_e * \Delta m$. The plot for the average and effective specific impulse is presented in figure 11. The maximum average specific impulse generated is 157s while the maximum effective specific impulse reached is 109s.
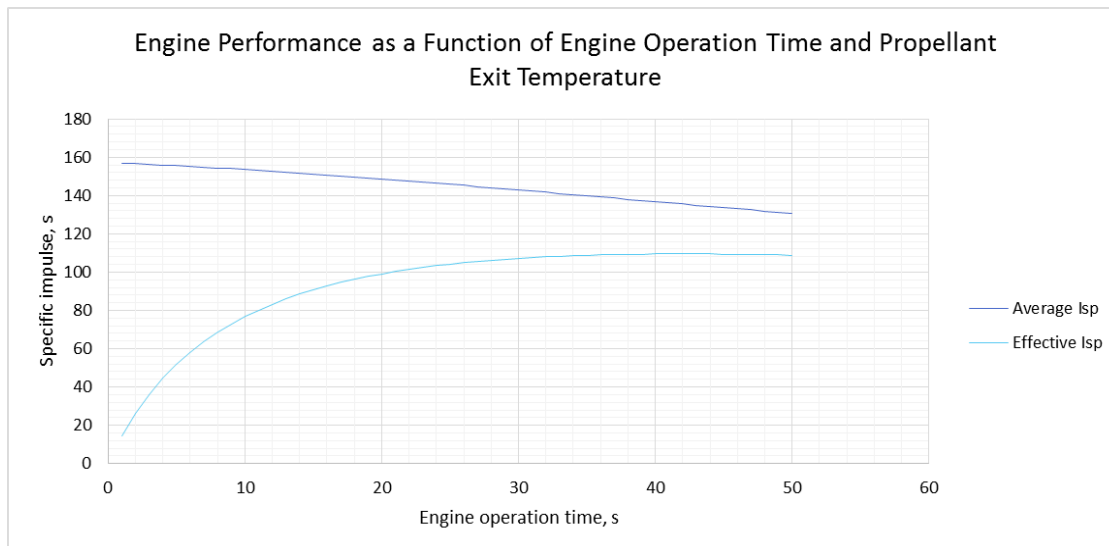


**Figure 11. Engine performance as a function of engine operation time and propellant exit temperature.**

## VI.  Tank Parametrics

### A.  Tank sizing

The Triton Hopper will use a spherical tank to store the nitrogen collected. The tank is assumed to be a COPV tank having a minimum thickness of 0.125in and constructed out of carbon fiber fabric which has a density of 1600 kg/m$^3$ and an ultimate tensile strength of 600MPa. The values for the density and ultimate tensile strength of the carbon fiber fabric were obtained from ACP composites.

The volume of the tank depends on the amount of nitrogen being stored, and the temperature and pressure at which it is being stored. Taking one of these parameters as a constant, allows for the determination of the tank's radius as a function of the other two parameters. Thus, by taking the mass of nitrogen as a constant 35kgs, the tank's size can be determined as a function of temperature and pressure.

The inner volume of the tank is determined from the ideal gas law. Choosing a factor of safety of 1.5 for the design, the maximum allowable stress is determined to be 400MPa. The thickness of the tank walls is determined from the hoop stress equation by solving for the thickness.

$$\sigma_{hoop} = SF * \frac{PD_{mean}}{4th} \tag{22}$$

Where SF is the safety factor chosen, P is the pressure inside the tank (Pa), *th* is the thickness of the tank (m), and D$_{mean}$ is the mean diameter of the tank (inner diameter plus the thickness). A plot of the size of the tank as a function of temperature, nitrogen mass and pressure is obtained by repeating these calculations for different values of mass, pressure and temperature.
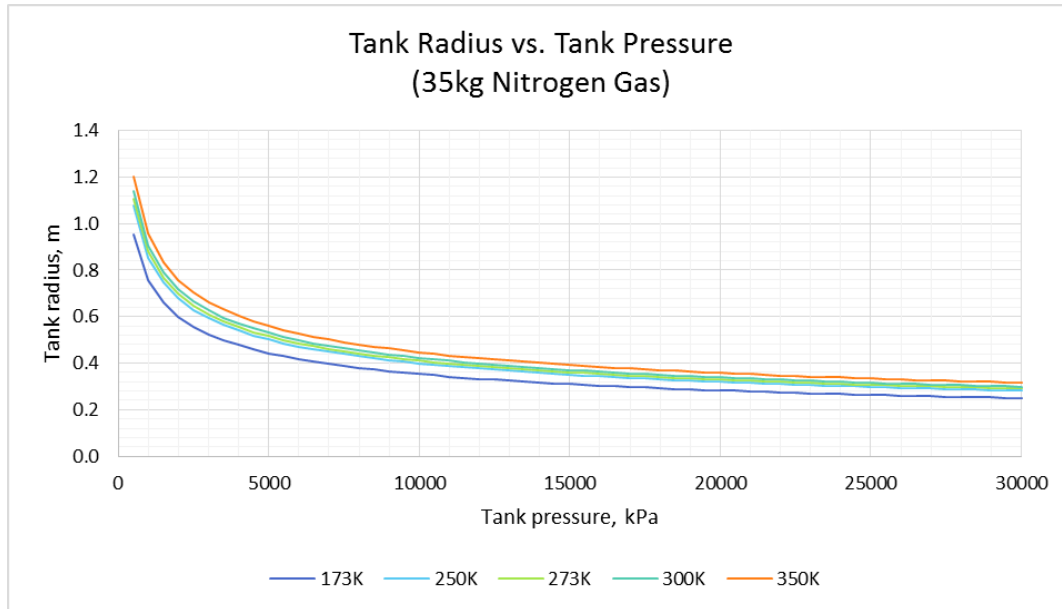


**Figure 12. Tank radius as a function of pressure and temperature.**

The mass of the tank is determined from equation 23, where P is the tank's pressure, V is the tank's volume, σ is the ultimate tensile strength of the material, and µ is the density of the material. Figure 13, presents a plot of the tank mass as a function of tank pressure and initial nitrogen temperature.

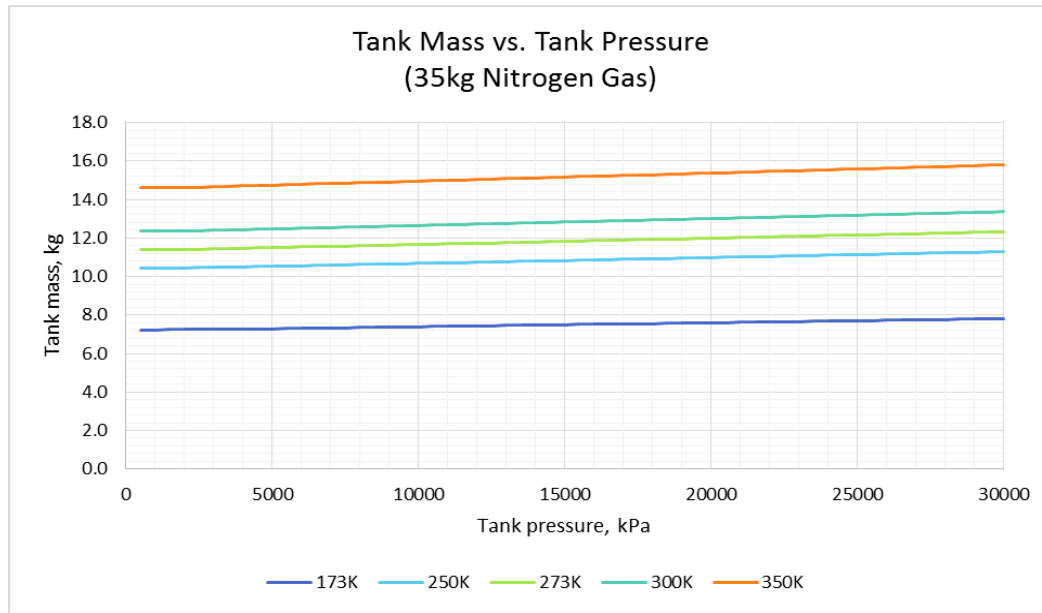$$M = \frac{3}{2}PV\frac{\sigma}{\mu} \tag{23}$$

**Figure 13. Tank mass as a function of pressure and temperature.**

## B. Tank insulation

The amount of heat that the system loses to its surroundings needs to be calculated in order to determine the amount of heat required to store the gas at the desired temperature and to heat the thermal block to 900°C. Multi-Layer Insulation (MLI) will be used to insulate the tank, thermal block, and any other component in the system that causes a high heat loss. The effective heat conductivity of MLI is determined from the effective heat conduction coefficient equation (24).

$$k_{eff} = 0.625 * \left( 1.027 * 10^{-7} * \left( \frac{T_H + T_C}{2} \right) + 3.333 * 10^{-16} * \left( \frac{T_H^{4.67} - T_C^{4.67}}{T_H - T_C} \right) \right) \tag{24}$$

Where $T_H$ is the temperature of the hot side and $T_C$ is the temperature of the cold side in Rankine. The values obtained from this formula have units of (Btu/hr)/ft-R, which means the need to be converted to SI (W/m-K).

The heat lost by the system is determined by solving simultaneously the equations of heat lost due to radiation and conduction. For the case of the nitrogen tank, the equations for heat lost due to conduction and radiation are:

$$\dot{Q}_{cond} = k_{eff} * \frac{A_s(T_H - T_s)}{th} \tag{25}$$

$$\dot{Q}_{rad} = \varepsilon * \sigma * A_s * \left( T_s^4 - T_{sur}^4 \right) \tag{26}$$

Where $\varepsilon$ is the emissivity of the insulation, $\sigma$ is Boltzmann's constant, $A_s$ is the surface area of the tank, $T_H$ is the temperature of the nitrogen inside the tank, $T_{sur}$ is the temperature of the surroundings, which is 34.5K for Triton's surface, and $T_s$ is the temperature of the outer surface of the insulation which is unknown. The surface temperature is determined by iterating for the value of $T_s$ until the heat lost due to conduction is the same as the heat lost due to radiation.

Analyzing the tank's heat loss, we have that the temperature of the hot side is 173K while the temperature of the cold side is 34.5K. From equation 24, the heat conduction coefficient for the MLI insulation is determined to be 0.0000214 W/m-K. Assuming an MLI insulation thickness of 10mm, the heat conduction and radiation equations are solved simultaneously for the surface temperature, which gives a heat loss of 0.615W when the surface temperature is 49.6K.

For the thermal mass, we have that the temperature of the hot side is 1173K while the temperature of the cold side is 34.5K. From equation 24, the heat conduction coefficient for the MLI insulation is determined to be 0.0007107 W/m-K. Assuming an MLI insulation thickness of 10mm, the heat conduction and radiation equations are solved simultaneously for the surface temperature, which gives a heat loss of 8.03W when the surface temperature is 187K.

In addition to the heat lost by the tank and the thermal block, heat is lost due to heat leaks such as the tubes that connect the tank, thermal mass and nozzle, which are non-insulated. These heat leaks need to be accounted for in order to determine the amount of heat energy required by the system. The heat lost from the tubes through radiation is equal to the heat conducted from the hot end to the cold end. For the tube connecting the tank and the heated block, the hot end is 900°C and the cold end is -100°C. Using titanium tubes of 1mm in diameter and 20cm in length, the heat lost is given by equation 25.

$$\dot{Q} = k * A_C * \frac{T_H - T_C}{L}$$
$$= 15.9\frac{W}{mK} * 0.0000817m^2 * \frac{1173K - 173K}{0.2m} = 6.49W$$

The heat loss for the tube connecting the thermal block and the nozzle is greater due to the fact that the nozzle is at ambient temperature.

$$\dot{Q} = k * A_C * \frac{T_H - T_C}{L}$$
$$= 15.9\frac{W}{mK} * 0.0000817m^2 * \frac{1173K - 34.5K}{0.2m} = 7.39W$$

The thermocouples used in the tank and the thermal block also count as heat leaks. A conservative calculation was done to determine the amount of heat lost if the thermocouples are made of Copper wire of 1mm and 1m length. The heat lost caused by the thermocouples is calculated from equation 25.

$$\dot{Q}_{tank} = k * A_C * \frac{T_H - T_C}{L}$$
$$= 385\frac{W}{mK} * 7.85 * 10^{-7}m^2 * \frac{173K - 34.5K}{1m} = 0.04W$$

$$\dot{Q}_{therm\_block} = k * A_C * \frac{T_H - T_C}{L}$$
$$= 385\frac{W}{mK} * 7.85 * 10^{-7}m^2 * \frac{1173K - 34.5K}{1m} = 0.34W$$

The results from the conservative analysis show the heat lost due to the thermocouples is negligible.

The total amount of heat lost from the system is 22.26W. The mission requirements indicate that the Triton Hopper must hop every 24 days. The power that needs to be provided by the RHUs or GPHS blocks is determined by taking the total amount of heat required to heat the thermal block material from -100°C to 900°C, subtracting the heat lost caused by heat leaks, and dividing the result by 24 days (in seconds). The results for the various thermal block materials are presented in figure 14.
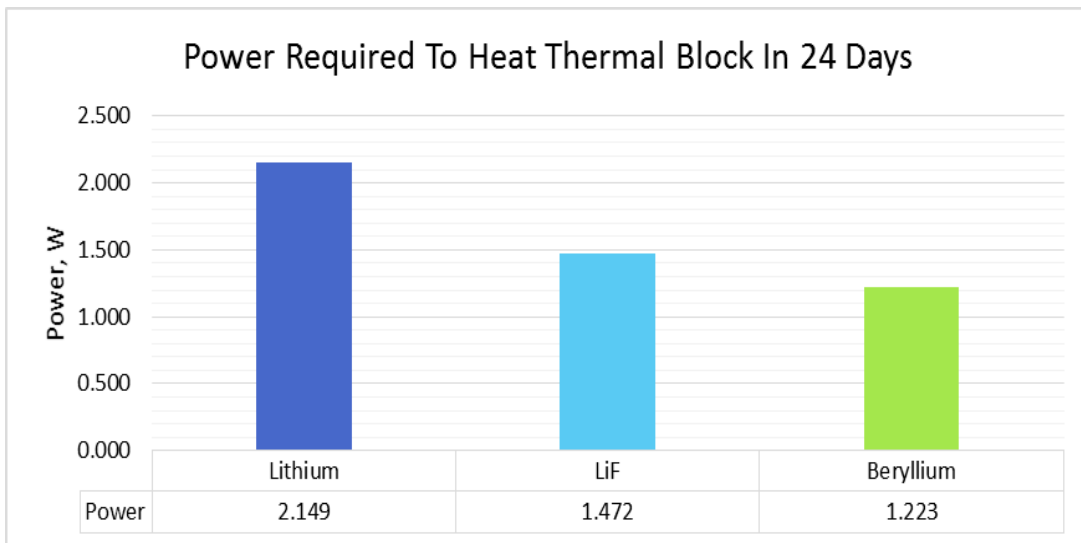
**Figure 14. Minimum power required to heat thermal block in 24 days.**

Knowing the minimum power required to heat the thermal block to 900⁰C in 24 days allows us to choose the heat source. RHUs provide 1W of power per unit, thus, 3 RHUs are enough to power the system. In order to reduce the time needed to heat the thermal block more RHUs can be added or they can be replaced with a GPHS block which provides 250W per unit. Figure 15 presents the days needed to heat the thermal block for two cases. Case 1 uses 25 RHUs and case 2 uses 1 GPHS block.
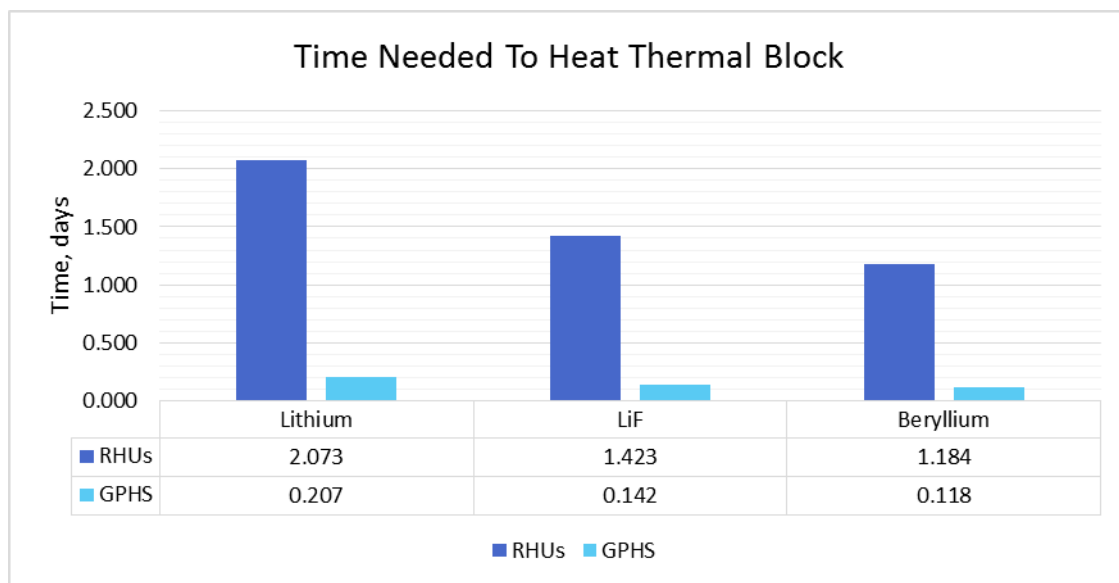


**Figure 15. Time required to heat thermal block.** *The figure shows the time needed to heat the thermal block for the case of using 25 RHUs or 1 GPHS block.*

## VII.  RTG Temperature Corrections

The analysis presented shows the results for an ideal case of where thermal mass reaches a temperature of 900°C. Realistically, RHUs would not be able to heat the thermal mass to this temperature. The Plutonium radioisotope source would reach a temperature of 1000°C, but the surface of the GPHS block which encapsulates the isotope, would only reach a temperature of 860°C. And the maximum temperature delivered to the hot side of the thermal mass would be approximately 760°C. Running the analysis with a thermal mass temperature of 760°C results in an average specific impulse of 147s and an effective specific impulse of 103s.
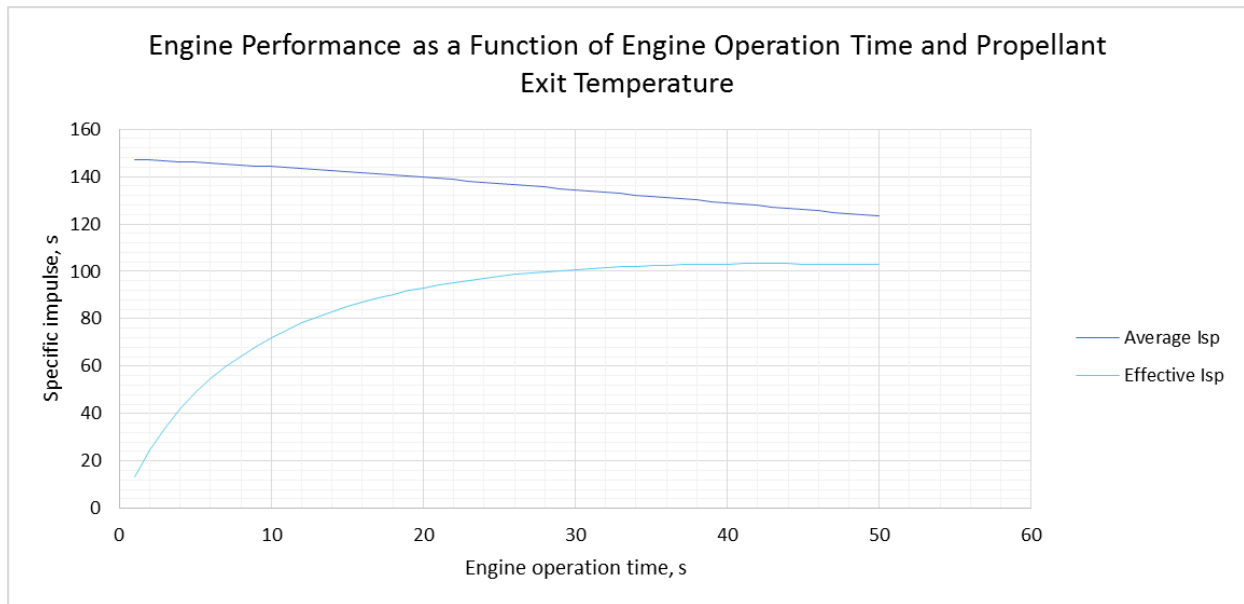


**Figure 16. Engine performance as a function of engine operation time and propellant exit temperature.** *The plot shows how the average and effective specific impulse of the engine decreases with longer engine operation time due to the decrease in thermal mass T.*

## VIII.  Conclusion

The results obtained from the specific impulse analysis indicate that the thermal block concept provides a considerable performance increase over the current concept. The estimated average specific impulse of the current system is 52s while the average specific impulse of the thermal block concept is 157s. Thus, heating the gas as the engine operates results in a performance increase of 200%. With the correction of the surface temperature of the thermal block, the specific impulse of the engine is reduced to 147s, which means the actual performance increase obtained is about 183%.  Additionally, the insulation and power analysis indicate that very little power is required to heat the thermal block to the operation temperature in a period of 24 days. For a lithium thermal mass, only 2.15 watts are required to heat it to 900°C. RHUs provide 1 watt per unit, which means only 3 units are required to power the system. For the case of the lower 760°C surface temperature, even less power is required. The low power requirement means that the time between hops can be reduced considerably by replacing the RHUs by a GPHS unit which provides 250 watts of power. The analysis also shows that the system can be sized to meet the mass requirements of the vehicle. A more detailed analysis of the engine needs to be performed but current results indicate that the thermal block radioisotope engine provides the best performance.

## Acknowledgments

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